

Risk Analysis Class Problem

Blanche has identified a risky venture opportunity. This venture will have one of two possible outcomes: It will be a flop, or it will be successful. If it is a flop, it will make \$0. If it is successful, it will make \$120,000. There is an 80% chance it will be a flop, and a 20% chance that it will be successful. It does not cost Blanche anything to undertake this venture.

Blanche has initial wealth of \$40,000, and a utility function

$$U(W) = W^{0.5}$$

where W is her wealth.

[Note: Carry all calculations to at least two decimal places in this problem]

(a) What is Blanche's certainty equivalent for this risky venture opportunity?

Blanche's certainty equivalent is: _____

Now Blanche hits upon another idea. She finds 10 members of her cohort, and decides to try to "sell" the opportunity to invest in this venture for \$2,000 per investor. Each of the 10 cohort members that Blanche approaches with this investment offer has an initial wealth of \$90,000, and a utility function $U(W) = W^{0.5}$, where W is the cohort member's wealth.

If a cohort member invests \$2,000 with Blanche, then:

- If the venture is successful, Blanche keeps the investor's \$2,000 and pays the cohort member 1/10th of the upside (that is, \$12,000). This means the investor enjoys a net gain of \$10,000, and a final wealth level of \$100,000.
- If the venture is a flop, however, then Blanche will keep the investor's \$2,000 and pay the investor nothing in return. The investor suffers a net loss of \$2,000, and a final wealth of \$88,000.

(b) I-Banker Joe is one of the ten cohort members Blanche approaches. What is Joe's certainty equivalent for the deal that Blanche is offering? If Joe's only two options are either to accept Blanche's offer, or to maintain his initial wealth of \$90,000 for sure, will Joe accept Blanche's offer?

Joe's certainty equivalent if he invests with Blanche is _____

Joe (will, will not) accept Blanche's offer because: _____

Circle One

- (c) Is Blanche better off selling-off her investment opportunity to 10 cohort members (under the terms described above), or is she better off taking the investment opportunity herself?

Blanche should (sell off the opportunity, take the opportunity herself)

Circle One

because: _____

Risk Analysis Class Problem Solutions

Part a: Blanche's expected utility if she invests is

$$\begin{aligned} E(U(W)) &= 0.8(40,000)^{0.5} + 0.2(40,000 + 120,000)^{0.5} \\ &= 0.8(40,000)^{0.5} + 0.2(160,000)^{0.5} \\ &= 0.8(200) + 0.2(400) \\ &= 160 + 80 = 240 \end{aligned}$$

The certainty equivalent (CE) is the expected utility squared, so

$$CE = [E(U(W))]^2 = (240)^2 = \$57,600$$

Part b: Joe's expected utility if he invests with Blanche is

$$\begin{aligned} E(U) &= 0.2(90,000 + 10,000)^{0.5} + 0.8(90,000 - 2,000)^{0.5} \\ &= 0.2(100,000)^{0.5} + 0.8(88,000)^{0.5} \\ &= 0.2(316.228) + 0.8(296.648) \\ &= 63.246 + 237.318 \\ &= 300.564 \end{aligned}$$

Joe's certainty equivalent for the deal Blanche is offering is his expected utility squared so

$$CE = [E(U(W))]^2 = (300.564)^2 = \$90,338.66$$

His expected utility if he does not invest with Blanche is

$$E(U) = (90,000)^{0.5} = 300$$

Since $300.564 > 300$, Joe will invest with Blanche. Another way of looking at it is that since Joe's CE from investing with Blanche is $\$90,338.66$ exceeds his CE from not investing with Blanche ($\$90,000$), he will invest.

Part c: If Blanche sells off the opportunity, her profit from doing so will be $10 \times 2,000 = 20,000$. Thus, her wealth grows from $\$40,000$ to $\$60,000$. This level of wealth gives her expected utility of $E(U(W)) = (60,000)^{0.5} = 244.949$. If she invests, her expected utility is 240. Since $244.949 > 240$, she is better off selling the opportunity. Another way of looking at it is that since Blanche's CE from selling off the opportunity to invest is $\$60,000$ exceeds her CE from investing ($\$57,600$), she will sell off the opportunity.

You must be careful with this problem. Blanche's expected wealth if she takes the investment herself is

$$E(W) = 0.8(40,000) + 0.2(160,000) = 32,000 + 32,000 = \$64,000$$

whereas her expected wealth if she sells her investment opportunity is

$$E(W) = 40,000 + 20,000 = \$60,000$$

Even though $\$64,000 > \$60,000$, it's not the expected wealth that governs her decision, it's the expected utility (because she's risk averse).